



Cambridge International AS & A Level

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MATHEMATICS

9709/31

Paper 3 Pure Mathematics 3

October/November 2024

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

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- 1 The polynomial $4x^3 + ax^2 + 5x + b$, where a and b are constants, is denoted by $p(x)$. It is given that $(2x+1)$ is a factor of $p(x)$. When $p(x)$ is divided by $(x-4)$ the remainder is equal to 3 times the remainder when $p(x)$ is divided by $(x-2)$.

Find the values of a and b .

[5]

This image shows a full page of a worksheet designed for handwriting practice. It features approximately 20 evenly spaced, horizontal dotted lines across the entire width of the page. The background is plain white, providing a clear guide for letter formation and alignment. There are no margins, text, or other markings present.



[5]

This image shows a full page of a document template. It consists of approximately 28 evenly spaced horizontal dotted lines across the entire width of the page, providing a guide for handwriting or typing. There are no margins, text, or other markings present.



3 The equation of a curve is $\ln(x+y) = 3x^2y$.

Find the gradient of the curve at the point $(1, 0)$.

[4]

[illegible]



4 (a) Show that $\sec^4 \theta - \tan^4 \theta \equiv 1 + 2 \tan^2 \theta$.

[3]

[illegible]



- (b)** Hence or otherwise solve the equation $\sec^4 2\alpha - \tan^4 2\alpha = 2 \tan^2 2\alpha \sec^2 2\alpha$ for $0^\circ < \alpha < 180^\circ$. [5]

[illegible]



- 5 (a)** By sketching a suitable pair of graphs, show that the equation $2 + e^{-0.2x} = \ln(1+x)$ has only one root. [2]

- (b) Show by calculation that this root lies between 7 and 9. [2]

[illegible]



(c) Use the iterative formula

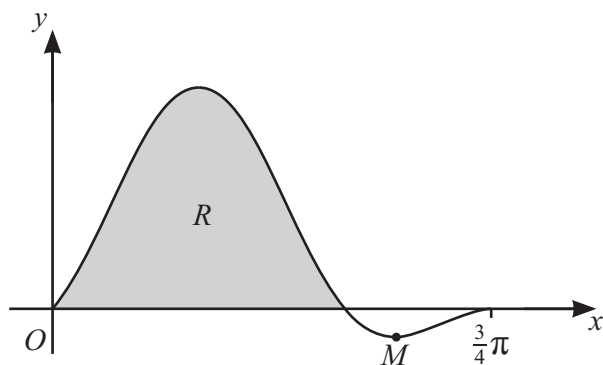
$$x_{n+1} = \exp(2 + e^{-0.2x_n}) - 1$$

to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places.

[$\exp(x)$ is an alternative notation for e^x .]

[3]

[illegible]



The diagram shows the curve $y = \sin 2x(1 + \sin 2x)$, for $0 \leq x \leq \frac{3}{4}\pi$, and its minimum point M . The shaded region bounded by the curve that lies above the x -axis and the x -axis itself is denoted by R .

- (a) Given that the x -coordinate of M lies in the interval $\frac{1}{2}\pi < x < \frac{3}{4}\pi$, find the exact coordinates of M . [4]

[illegible]



(b) Find the exact area of the region R .

[4]

[illegible]



Let $f(x) = \frac{5x^2 + 8x + 5}{(1 + 2x)(2 + x^2)}$.

[5]

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(b) Hence find the coefficient of x^3 in the expansion of $f(x)$.

[4]

[illegible]



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- (c) On a single Argand diagram, sketch the loci given by the equations $\operatorname{Re}(z) = 1$ and $\left|z - \frac{1}{2}\right| = \frac{1}{2}$, where z is a complex number. [3]

- (d) The complex number z is such that $\operatorname{Re}(z) = 1$. Use your answer to part (b) to give a geometrical description of the locus of $\frac{1}{z}$. [1]





- (a) State a vector equation for l .

[2]

- Find the value of t .

[3]

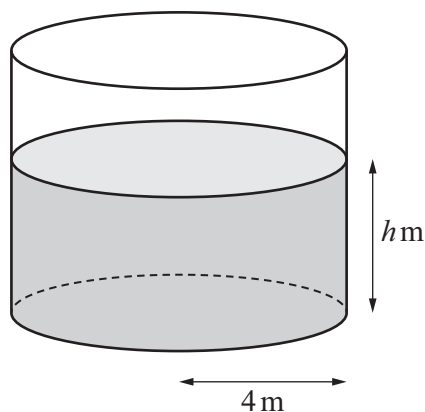


- (c) The line m has vector equation $\mathbf{r} = 5\mathbf{i} - \mathbf{j} + 2\mathbf{k} + \mu(a\mathbf{i} - \mathbf{j} + 3\mathbf{k})$. The acute angle between the directions of l and m is θ , where $\cos \theta = \frac{1}{\sqrt{6}}$.

Find the possible values of a .

[5]

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A large cylindrical tank is used to store water. The base of the tank is a circle of radius 4 metres. At time t minutes, the depth of the water in the tank is h metres. There is a tap at the bottom of the tank. When the tap is open, water flows out of the tank at a rate proportional to the square root of the volume of water in the tank.

- (a) Show that $\frac{dh}{dt} = -\lambda\sqrt{h}$, where λ is a positive constant. [4]

[illegible]



- (b)** At time $t = 0$ the tap is opened. It is given that $h = 4$ when $t = 0$ and that $h = 2.25$ when $t = 20$.

Solve the differential equation to obtain an expression for t in terms of h , and hence find the time taken to empty the tank. [6]

[illegible]



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